Horsing Around With $Z = X*Y$

Celebration of Mind
October 30, 2016

John Partridge
jp@pyrigan.com

Some Context

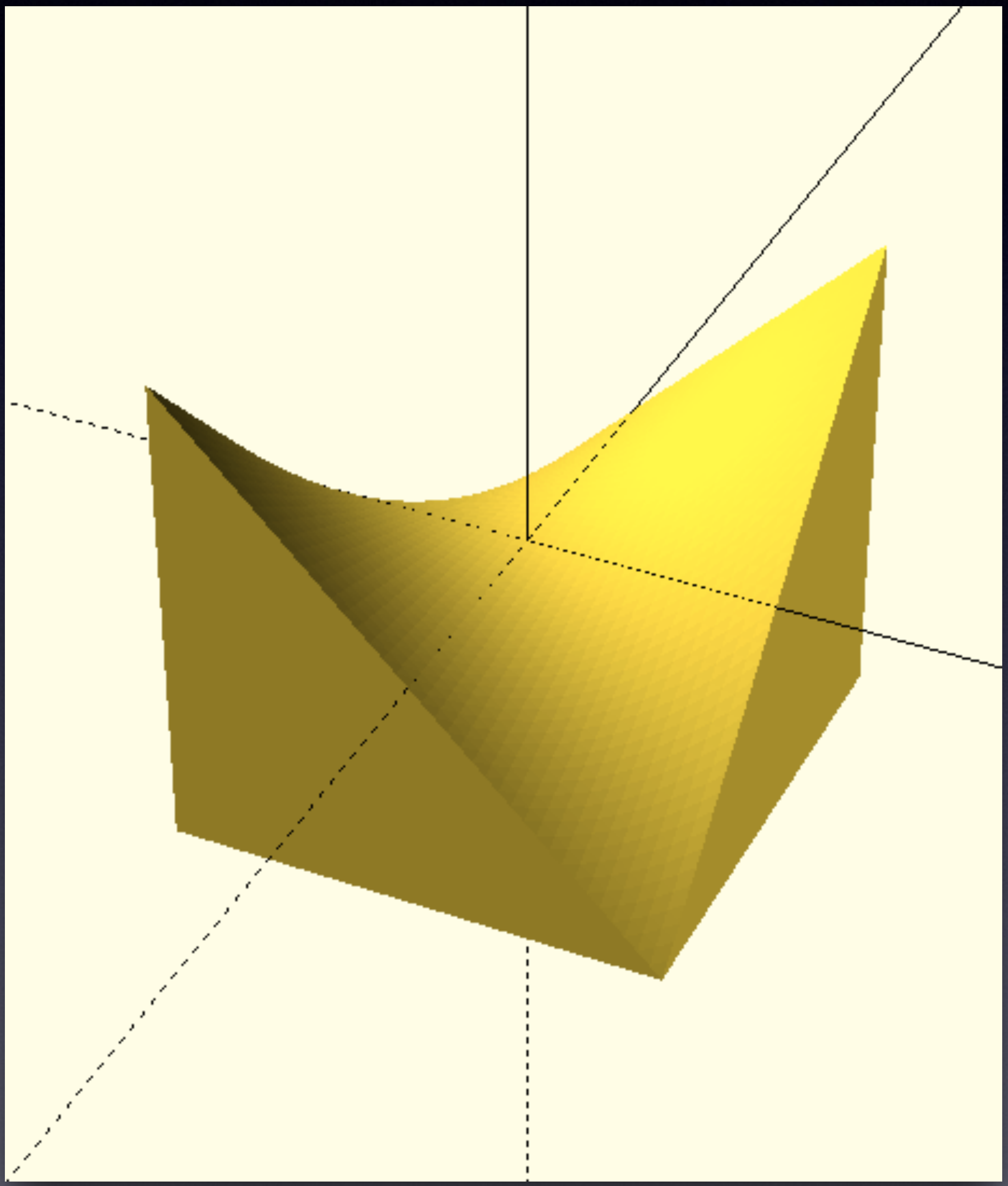
- Subtitle: "*What happens when a non-mathematician tries to visualize an hyperbolic paraboloid on the cheap*"

- I'm a software guy / high tech startup executive
  - Grew up reading Scientific American
  - Built stuff in Fischer-Technik
  - Learned to program on a PDP-8/E

Spheres and Hyperbolae

- A sphere's curvature has a constant positive value at all points on its surface. OK, fine.
- But what's the shape whose curvature is a constant *negative* value at all points on its surface?
- Simplest answer I could find was

  Z = X*Y



But how do I visualize that?

- OpenSCAD to the rescue!
  - **`http://www.openscad.org/`**

- CAD tool that lets you design shapes by programming in a procedural language

- OpenSCAD demo, Part I

Visualization Feeding Frenzy

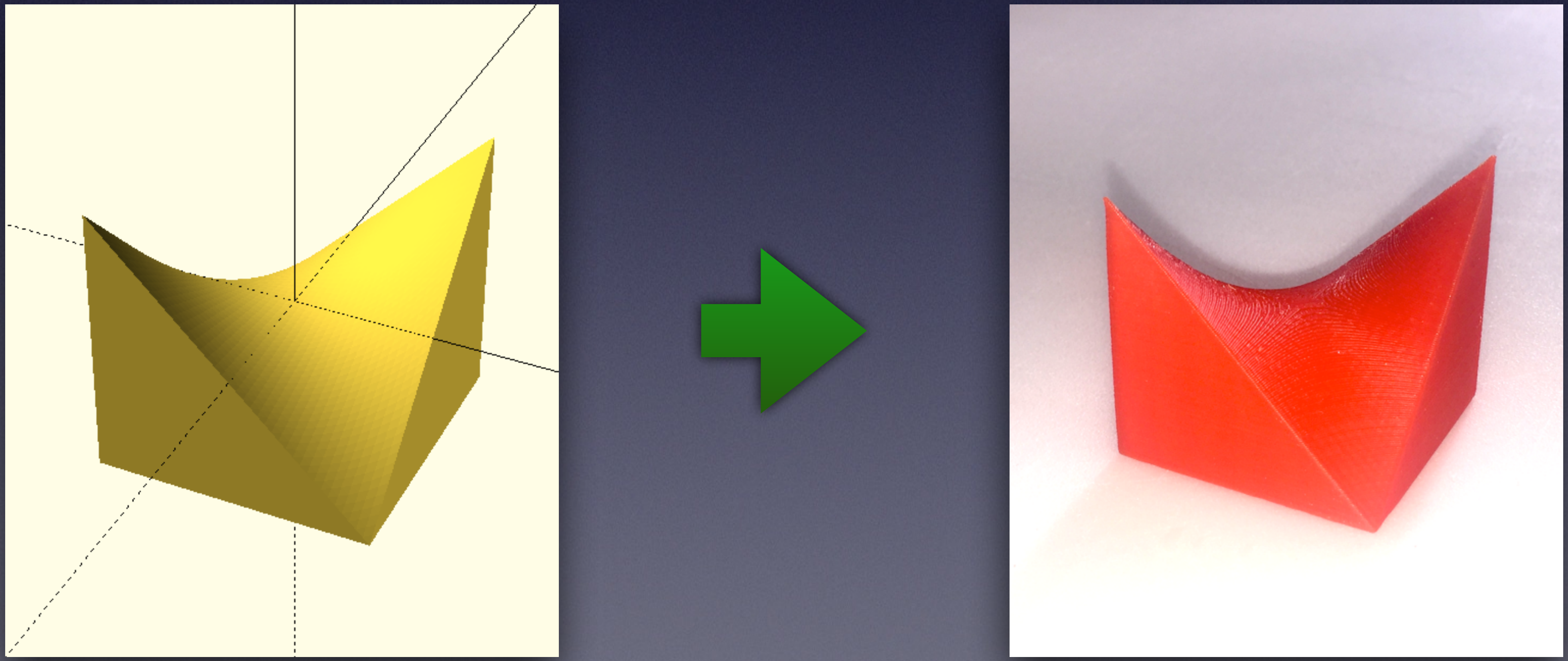
- How do those diagonal edges line up?
- Do they make seams or do they make a continuous surface?
- As I add unit hyperbolae in the X, Y, and Z directions, do they eventually interfere?

What does it look like?

- OpenSCAD demo, Part II

What If Visualization Just Isn't Enough?

- 3D Printing to the rescue!
  - **`https://www.makerfront.com/`**



Feeding Frenzy ++

- Since this shape is composed of copies of the same hyperbola and each is its own spatial inverse, this shape must be its own inverse, mustn't it?
- It has triangular radial symmetry at two of the corners; what does dissecting it into thirds look like?
- I know I can tile space with those weird shapes but can the pieces be assembled / joined without deformation?

Time for Show and Tell

Nagging Questions

- Is the 4x4x4's corner hole a perfect circle?
- Is the 4x4x4 cube the smallest cube possible for repeating this three dimensional pattern?
- Or is there some rotation or translation of a smaller sample?
- What is the largest shape that when assembled with copies of itself, produces this three dimensional pattern and assembles without deformation?
- I'm hungry; is this the last slide?

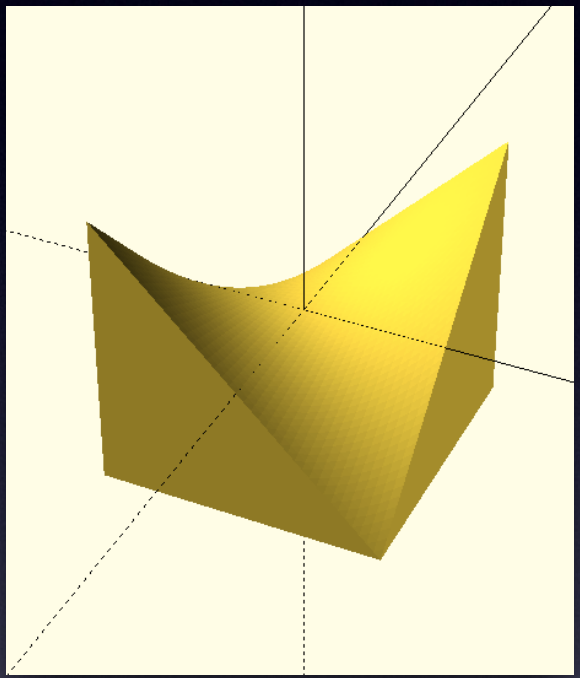
Yes!

Celebration of Mind
October 30, 2016

John Partridge
jp@pyrigan.com

Appendix I:
OpenSCAD Code for Unit Hyperbola

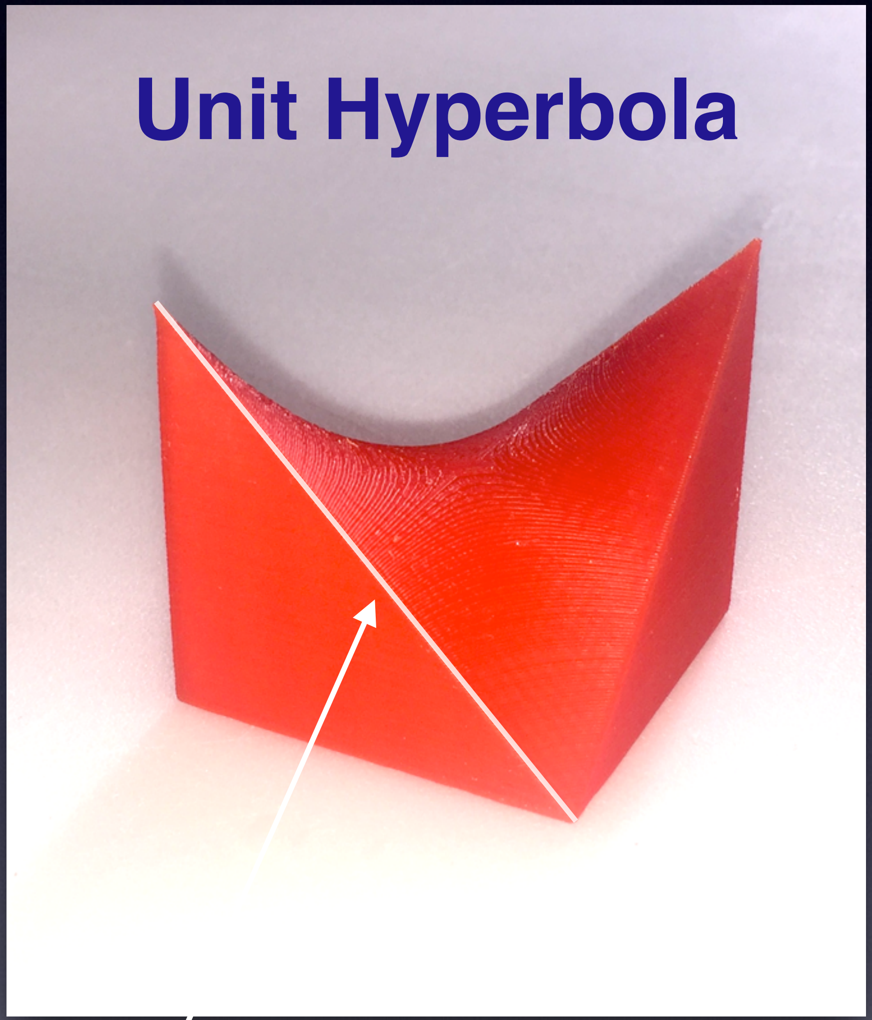
```
//
// OpenSCAD for my "Horsing Around with Z=X*Y" talk
// John Partridge 10/31/2016
//

xmin = -1;
xmax = 1;

ymin = -1;
ymax = 1;

zmin = min(xmin * ymax, xmax * ymin);

step = .05; // a value of .05 is very high resolution

xpoints = floor((xmax - xmin) / step) + 1;
ypoints = floor((ymax - ymin) / step) + 1;
function index(x,y) = x * ypoints + y + 2; // plus 2 because of two extra points added to make the base
function x(i) = xmin + i * step;
function y(i) = ymin + i * step;

points = concat([[xmin, ymin, zmin]], // two extra points to allow a rectangular base
                [[xmax, ymax, zmin]],
                [for(i = [0 : xpoints - 1], j = [0 : ypoints - 1]) [x(i), y(j), x(i) * y(j)]]);

base = concat(
        [[1, 0, index(xpoints - 1, 0)]], // two triangles to make the base
        [[0, 1, index(0, ypoints - 1)]],
        // four triangule fans for the sides
        [for(i = [0 : xpoints - 2]) [0, index(i, 0), index(i + 1, 0)]],
        [for(i = [0 : xpoints - 2]) [1, index(i + 1, ypoints - 1), index(i, ypoints - 1)]],
        [for(j = [0 : ypoints - 2]) [0, index(0, j + 1), index(0, j)]],
        [for(j = [0 : ypoints - 2]) [1, index(xpoints - 1, j), index(xpoints - 1, j + 1)]]
       );

faces = concat( // connect every four points with two triangles
        [for(i = [0 : xpoints - 2], j = [0 : ypoints - 2]) [index(i,j), index(i, j + 1), index(i + 1, j + 1)]],
        [for(i = [0 : xpoints - 2], j = [0 : ypoints - 2]) [index(i + 1, j + 1), index(i + 1, j ), index(i, j)]]
        );


polyhedron(points, concat(base, faces), convexity=2);
```



Appendix II:
Show and Tell Pictures

Recipe:

1. Make a bunch of copies of the unit hyperbola (care to guess how many you'll need?)
2. Join a pair of them such that the diagonal edge on one piece aligns continuously and seamlessly with the diagonal edge on the other piece
3. Join them all together this way until you have (half of) a 4x4x4 cube:

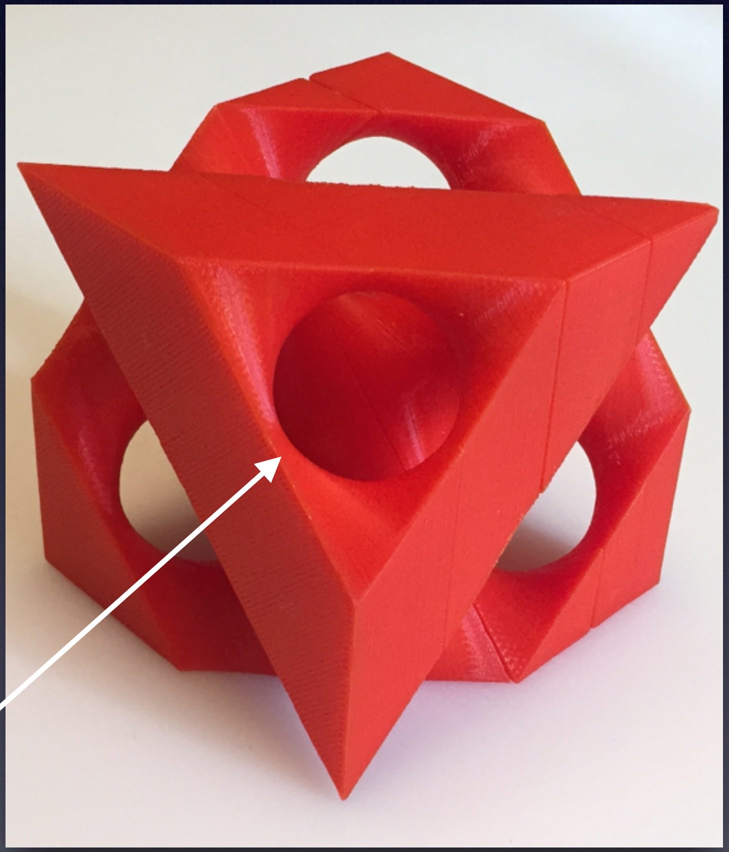


Diagonal

Appendix II:
Show and Tell Pictures

If you rotate and tilt this:

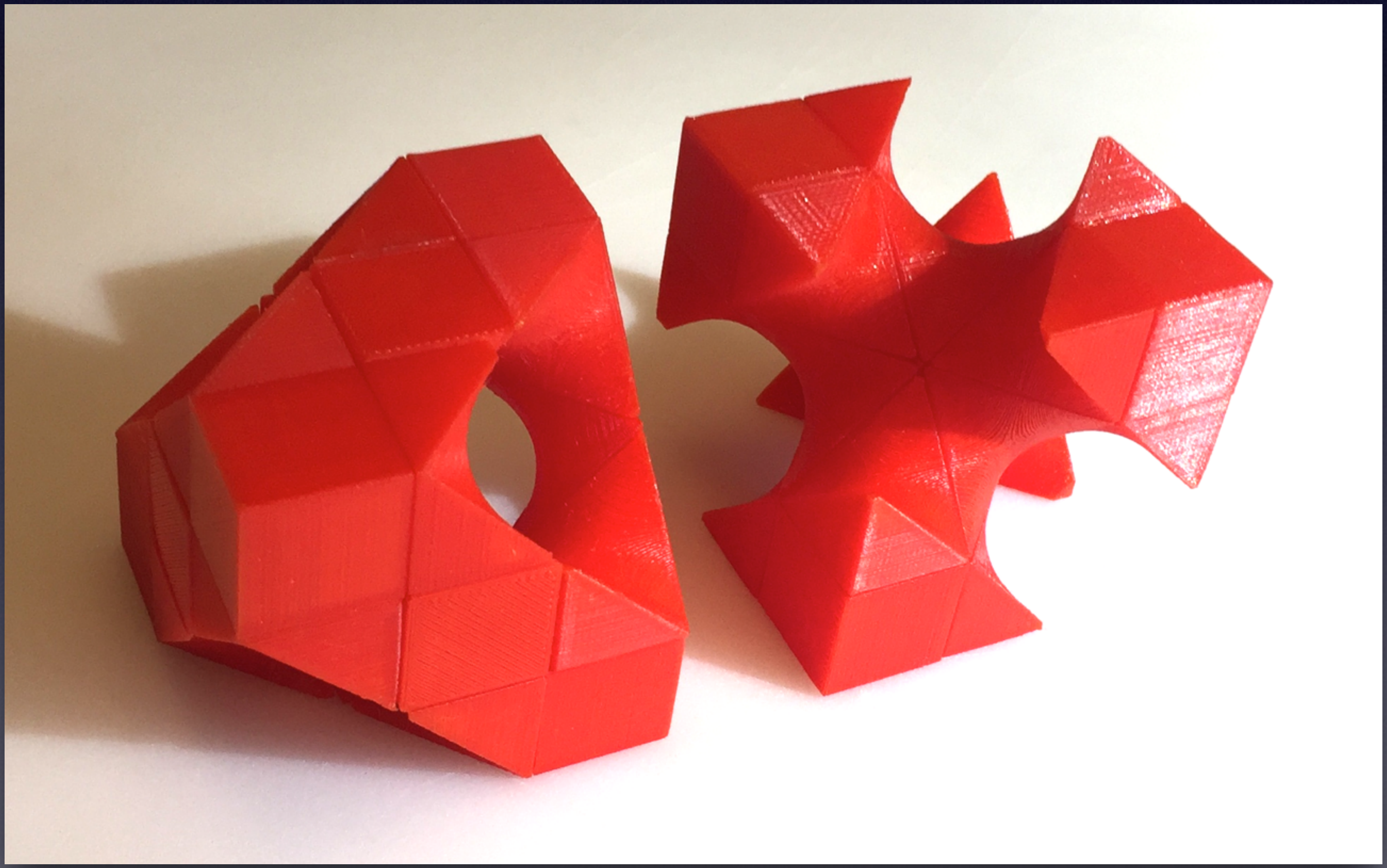
You'll see this:



Is the hole a perfect circle or merely circular?

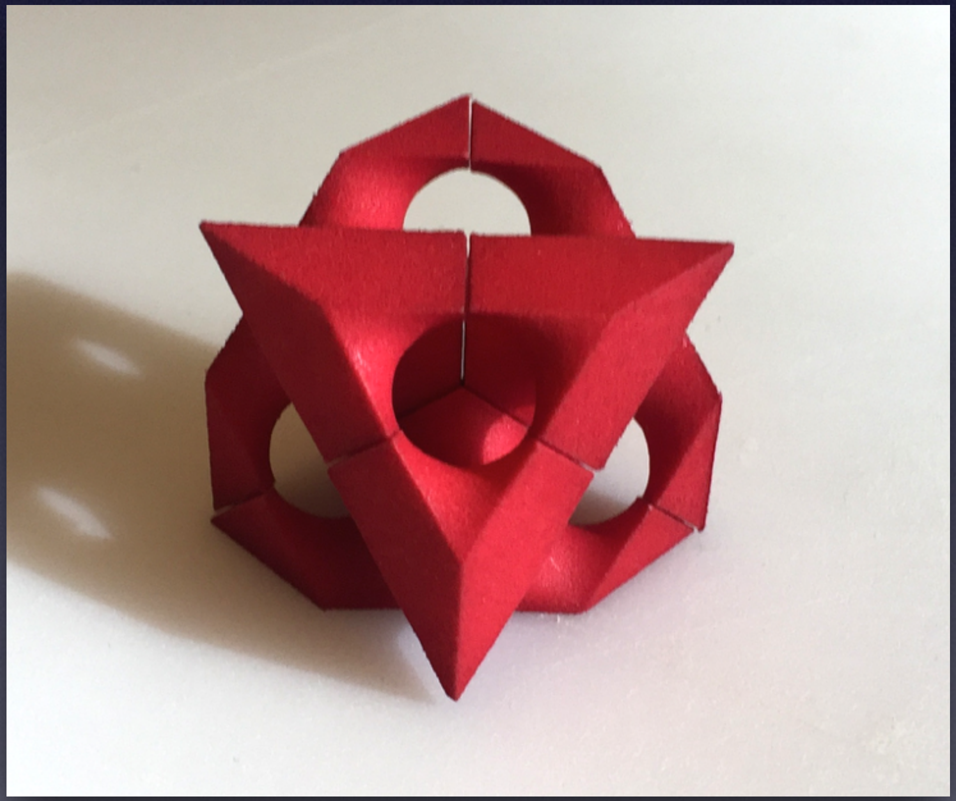
Appendix II: Show and Tell Pictures

Here are what two 3x3x3 assemblies look like (they are inverses of each other):

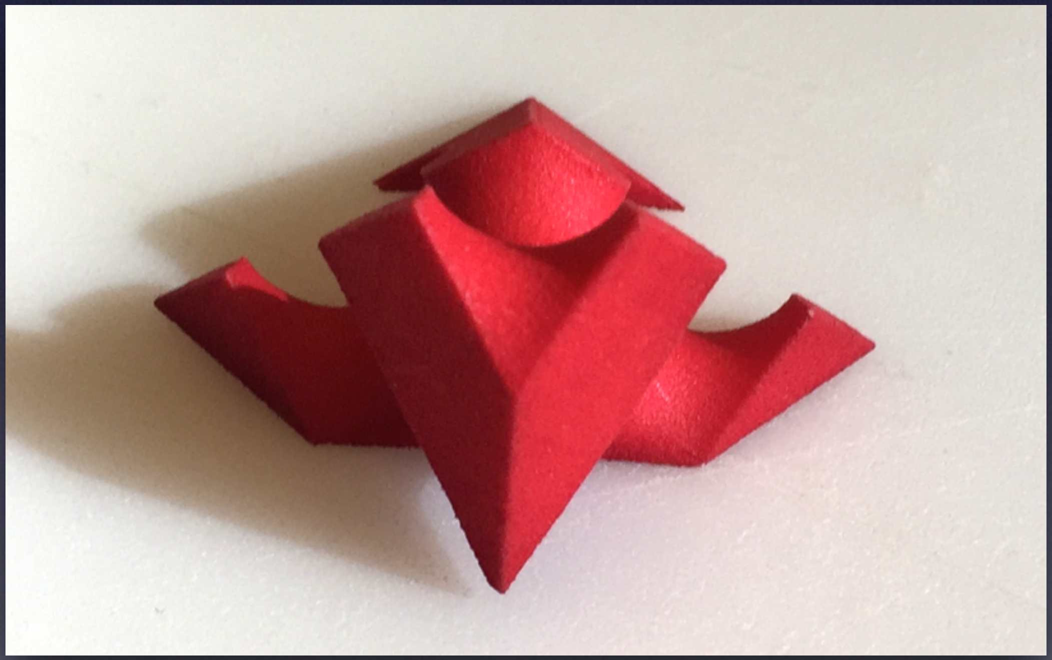


Appendix II:
Show and Tell Pictures

Let's cut the 4x4x4 assembly into thirds:

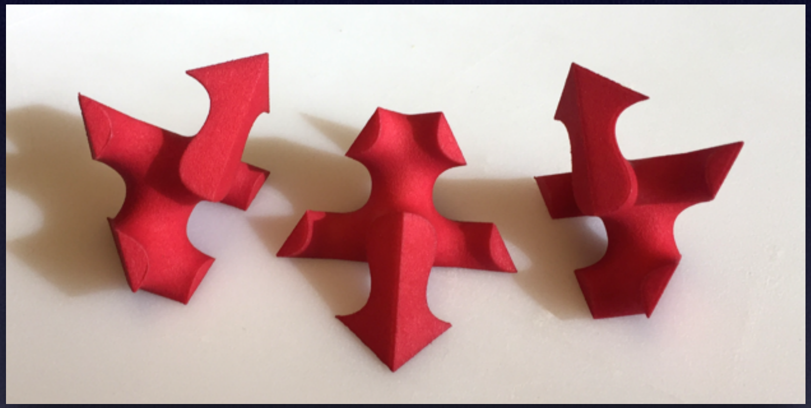


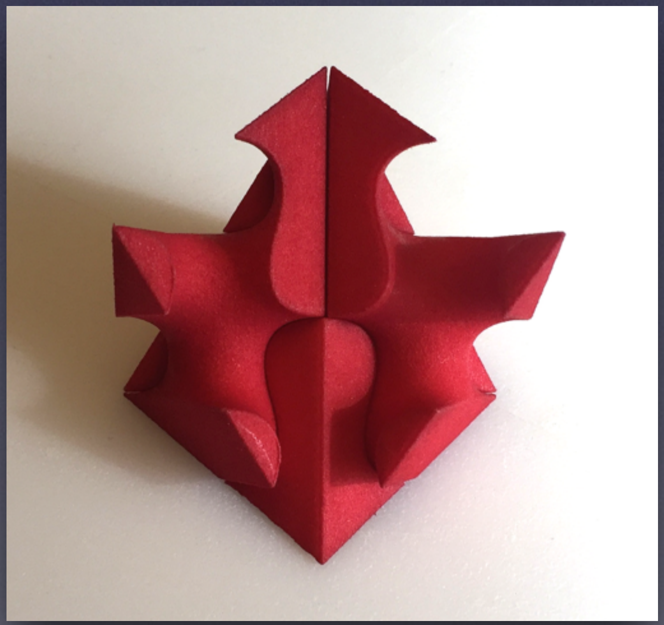
÷ 3 =

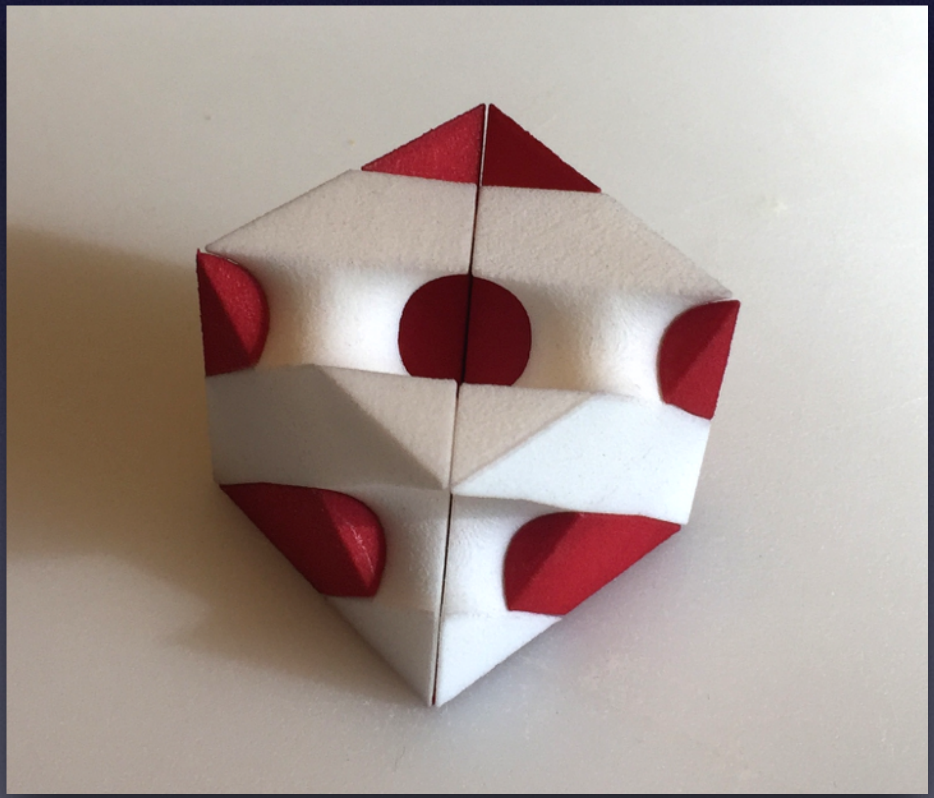


Appendix II:
Show and Tell Pictures

Now we'll reassemble the pieces and add some white ones:







Appendix II:
Show and Tell Pictures

Finally, let's rearrange the pieces to make a striped cube: